

Using Syllogistics to Teach Metalogic

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Abstract

This paper describes a specific pedagogical context for an advanced logic course, and presents a strategy that might facilitate students' transition from the object- to the metatheoretical perspective on logic. The pedagogical context consists of philosophy students who have in general had little training in logic, except for a thorough introduction to syllogistics. The teaching strategy tries to exploit this knowledge of syllogistics, by emphasizing the analogies between ideas from metalogic and ideas from syllogistics, such as existential import, the distinction between contradictories and contraries, and the square of opposition. This strategy helps to improve the students' understanding of metalogic, because it allows them to integrate these new ideas with their previously acquired knowledge of syllogistics.

Keywords: teaching logic, metalogic, syllogistics, square of opposition.

1 Introduction

Although logic has a strong historical connection to philosophy, it is nowadays widely acknowledged that logic has become a highly interdisciplinary research area, with applications and interactions in diverse fields such as mathematics, computer science, psychology, law and linguistics. From an educational perspective, this interdisciplinary nature implies that logic courses are being taught (at an introductory as well as a more advanced level) to students coming from all of these fields, each of them approaching the course with their own individual educational backgrounds, attitudes and expectations about logic. Furthermore, throughout higher education, it is a good practice to teach new material in such a way that it directly *builds upon* students' previously acquired skills and knowledge, since this method seems to maximize students' comprehension and retention of the new material (Hativa 2000, 56ff.). As a consequence, the most efficient way to teach logic can vary greatly from one target audience to another. For example, an efficient introductory logic course for computer science students will typically make

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use of different analogies, motivating examples, types of exercises, etc. than an equally efficient introductory logic course for philosophy students.

Another issue in teaching logic stems from the sharp discontinuity in the perceived degree of difficulty between ‘object-level logic’ and ‘meta-level logic’. The former is typically concerned with assessing the deductive validity of arguments using some standard logical systems, and involves translating natural language arguments into first-order formulas, truth tables, tableaux, natural deduction, etc. The latter is more concerned with the mathematical properties of the logical systems themselves, and involves notions such as soundness, completeness, compactness, models of various infinite cardinalities, etc. Object-level logic is typically taught in a first, introductory course on logic, while metatheory is generally discussed in detail only in subsequent, more advanced courses. Although the introductory course is usually among the formal prerequisites for the more advanced courses, many students seem to find the latter disproportionately harder than the former—for example, nearly every logic teacher will have encountered students who performed excellently in their introductory logic course, but then went on to struggle in more advanced logic courses.¹ At least a partial explanation for this widespread phenomenon is that some students might fail to efficiently process the new, metalogical ideas, precisely because they are insufficiently able to integrate them with—and view them as *building upon*—their previously acquired knowledge of object-level logic.

In this paper, I will describe a specific pedagogical context for an advanced logic course in terms of the students’ educational background and attitudes, and present a teaching strategy that might facilitate students’ transition from the object- to the metatheoretical perspective on logic. The pedagogical context consists of a highly heterogeneous audience of philosophy students, who have in general had relatively little training in logic, except for the fact that a large majority of them have had a thorough introduction to classical Aristotelian logic, i.e. *sylogistics*. The teaching strategy tries to exploit this knowledge of sylogistics, by emphasizing certain analogies between ideas from metalogic and ideas from sylogistics, such as existential import, the distinction between contradictories and contraries, and the square of opposition. This strategy helps to improve the students’ understanding of metalogic, because it allows them to integrate these new ideas with their previously acquired knowledge of sylogistics.

The paper is organized as follows. Section 2 delineates a specific type of audience for an advanced logic course, and explains why so many of these students have a thorough knowledge of sylogistics. Next, Section 3 describes the concrete

¹I do not mean to deny, of course, that introductory logic courses come with their own set of specific pedagogical challenges. Some students have a hard time adapting to the use of mathematical symbolism to represent and analyze natural language arguments, and it has been argued that it might be more efficient to postpone the introduction of mathematical symbolism longer than is usually done (Griffith 1975; Carrascal 2011). These issues can become quite thorny, for example when an otherwise well-performing student with a specific learning disability that hampers her capacities for mathematical thinking asks for a waiver of her required logic course (Faust 2007).

metalogic/syllogistics analogies that can be used in the classroom, and offers some reflections on their wider theoretical significance. Section 4 then illustrates the efficacy of these analogies, by showing how they can help students to understand some notoriously confusing ideas and theorems from metalogic. Finally, Section 5 wraps things up, and suggests some topics for further exploration.

2 An Educational Context for an Advanced Logic Course

In this section, I will describe a specific context for an advanced logic course, and trace some of the broad historical-cultural factors that have probably played a role in how it came about. The description of this context is primarily based on my personal experiences with teaching such a course at a single institution (viz. the Institute of Philosophy at KU Leuven, Belgium), but because of the wide-ranging historical-cultural factors involved in its evolution, one is likely to encounter highly similar contexts at many other institutions as well.

At KU Leuven, the course under discussion is taught in the master (MA) program in philosophy. It is officially entitled ‘Logic: Advanced Course’, and the course contents are meant to live up to this title. After a few weeks of introductory classes, we dive into abstract metatheoretical issues, such as consistency, soundness, completeness, compactness, etc. (these theorems are presented with a detailed proof in the case of propositional logic and modal logic, and a proof sketch in the case of predicate logic). The most recent editions of the course were attended by 15 to 30 students, coming from a variety of different backgrounds. Some students have previously obtained a bachelor’s (BA) degree in philosophy at KU Leuven itself, and some are ‘visitors’ from other MA programs at KU Leuven, such as statistics, computer science and physics (and thus typically have a BA degree in one of these fields). The large majority, however, consists of ‘incoming’ students, who come from places all over the world. Many of them already have a significant background in philosophy, and typically come to Leuven with the explicit intention of pursuing a PhD in areas such as phenomenology and ancient and medieval philosophy (because of the presence of strong research units on these topics in Leuven). Finally, some international students are in Leuven as part of their priest training, and next to their core theology courses, they can take a number of courses at the Institute of Philosophy, including the advanced logic course.

The heterogeneity of this audience is manifested in multiple ways. Students typically enter the course with widely different attitudes toward the discipline of logic, ranging from mild enthusiasm over indifference to outright hostility (“formal logic has nothing to teach us about the human condition!”).² Similarly, there

²In this paper, I will not enter into the discussion of the value and/or necessity of a formal logic course in the philosophy curriculum. In the past decades, a variety of reasons has been given for teaching formal logic to philosophy students, ranging from the practical (logic is indispensable to be able to read papers in contemporary analytic philosophy) to the humanistic (logic is a liberal art that is valuable in itself) (Massey 1981; Van Evra 1985; Decker 2011).

are huge differences in students' earlier education in logic, ranging from quite extensive to virtually non-existent (e.g. never having seen truth tables for propositional logic). One important exception, however, is that nearly all students have had a relatively thorough introduction to Aristotelian logic, i.e. *sylogistics*. At first sight, this might seem strange, since the received view holds that syllogistics is rarely taught nowadays, especially in analytically oriented philosophy departments in the US, the UK and (more recently) continental Europe.³ Now, the introductory logic course in the Leuven philosophy BA program still contains an introduction to syllogistics, which straightforwardly (but idiosyncratically) explains why students from Leuven know about it. However, to account for the incoming students, some broader historical-cultural factors have to be taken into account.

Jaspers and Seuren (forthcoming) argue convincingly that, *pace* the received view, syllogistics managed to survive in one specific intellectual climate, viz. institutions associated with the Catholic church. They situate this within the more general Catholic cultural (neo-Thomistic) revival that occurred roughly between 1840 and 1960,⁴ but further historical analysis need not concern us here. At these Catholic institutions, syllogistics (and, in particular, the Aristotelian square of opposition) remained an important topic of active research, and thus also continued to be taught extensively.⁵ From this perspective, the widespread knowledge of syllogistics among incoming students no longer seems strange, since these students typically come from institutions belonging to this Catholic intellectual climate. For example, many students coming to Leuven to pursue a PhD in phenomenology come from Catholic universities and colleges in North America. Furthermore, unsurprisingly, most students who are in Leuven as part of their priest training come from Catholic colleges and seminaries (especially in Africa and Southeast Asia).

3 Analogies between Metalogic and Syllogistics

In the previous section, I described a specific context for an advanced logic course, which involves students with extensive previously acquired knowledge of syllo-

³Here are two typical expressions of this received view: "If it is taught at all, Aristotelian syllogistic appears as no more than a curious footnote, perhaps a short chapter, in contemporary logic textbooks." (Oderberg 2005, viii); "the old syllogistic logic was not immortal. Though bits of it survive in places Peter Geach unfairly calls "Colleges of Unreason", it is, essentially, dead" (Engelbretsen 1990, 150).

⁴From this perspective, it seems much less idiosyncratic that syllogistics is still taught in the introductory logic course at the Institute of Philosophy in Leuven: the Institute was founded in 1889 by Cardinal Mercier under the authority of pope Leo XIII, and was one of the strongholds of neo-Thomism—it was unofficially referred to as the 'St Thomas School' until well into the 20th century (Struyker Boudier 1989).

⁵Authors belonging to this tradition were very explicit about this. For example, Doyle (1952, 93) claims that the "square of opposition has for a long time helped students understand some of the formal relationships that may obtain between pairs of propositions, and it is likely to be used with beneficial results for a long time to come.", while Hacker (1975, 352) introduces a new, octagonal extension of the square and states that he has "used the octagon of opposition for the past seven years in teaching Aristotelian logic and [...] found it a useful pedagogical device".

gistics. This knowledge can be put to good use when teaching metalogic to these students, by emphasizing certain conceptual analogies between syllogistics and metalogic. In this section, I will describe some of these analogies, and make some remarks about their wider theoretical significance.⁶

We start by considering the semantic notion of a formula φ being a *tautology* in a logical system S . Formally, φ is said to be a tautology in S (notation: $\models_S \varphi$) iff φ is true in all models of S . From the perspective of syllogistics, this definition is a categorical statement of universal quantity and affirmative quality: it is of the form “all S are P ”, with subject term S = “model of the logical system S ” and predicate term P = “makes φ true” (Parsons 2012). Next, we consider the semantic notion of *satisfiability*. A formula φ is said to be satisfiable in S (notation: $\not\models_S \neg\varphi$) iff φ is true in at least one model of S . This definition turns out to be a categorical statement of particular quantity and affirmative quality: it is of the form “some S are P ”, with the same subject term S and predicate term P as above.⁷ Let’s now see what happens if we switch the quality parameter in these categorical statements from affirmative to negative (while leaving the subject and predicate terms unchanged). The resulting universal negative statement is “all models of S do not make φ true”, which means the same as “no models of S make φ true”, and thus corresponds exactly to the semantic notion of *contradiction* (notation: $\models_S \neg\varphi$). Finally, the particular negative statement is “some models of S do not make φ true”, or equivalently, “not all models of S make φ true”; unlike the previous three statements, this one does not seem to correspond immediately to any standard, ‘primitive’ semantic notion, but it can be described as a *non-tautology* (notation: $\not\models_S \varphi$).

We have now obtained the four categorical statements that are typically used in syllogistics to construct a ‘square of opposition’. The table below summarizes these four statements, together with their corresponding metalogical notions and their well-known mnemonic abbreviations (from the first two vowels in the Latin verb forms ‘affirmo’ and ‘nego’).

| quantity | quality | letter | definition | notion | in symbols |
|------------|-------------|--------|------------------------------------|---------------|-----------------------------|
| universal | affirmative | A | all models make φ true | tautology | $\models_S \varphi$ |
| particular | affirmative | I | some models make φ true | satisfiable | $\not\models_S \neg\varphi$ |
| universal | negative | E | no models make φ true | contradiction | $\models_S \neg\varphi$ |
| particular | negative | O | not all models make φ true | non-tautology | $\not\models_S \varphi$ |

⁶A more systematic discussion of these analogies can be found in Béziau (2012; 2013), Demey and Smessaert (2016a) and Demey (2017a). Interestingly, all these authors focus exclusively on analogies with the *semantic/model-theoretical* aspects of metalogic (\models), and I myself will do the same in the remainder of this article. However, an anonymous reviewer has suggested that it might also be interesting to search for analogies with the *syntactic/proof-theoretical* aspects of metalogic (\vdash), and that taking both perspectives into account might also shed new (pedagogical) light on the soundness and completeness theorems. I am grateful for these interesting suggestions, and am exploring them in ongoing work (Demey 2017b).

⁷The notation $\not\models_S \neg\varphi$ suggests the following formulation: “not all models of S do not make φ true”, but this is of course equivalent to the more natural-sounding “some models of S make φ true”.

Before moving on, I would like to point out that the fact that the O-statement does not correspond to a widely used semantic notion does not have any negative repercussions for the analogy between syllogistics and metalogic in general. In fact, this discrepancy between the A-, I- and E-notions on the one hand and the O-notion on the other is entirely to be expected, since it is perfectly in line with previous, empirical work on natural language decorations of the square. Linguists have found exactly the same discrepancy in various closed lexical fields, such as the quantifiers (A: *all*, I: *some*, E: *no* versus O: **nall*), the propositional connectives (A: *and*, I: *or*, E: *nor* versus O: **nand*) and the temporal adverbs (A: *always*, I: *sometimes*, E: *never* versus O: **nalways*).⁸ Horn (1989; 2012) has developed a pragmatic (neo-Gricean) explanation for the systematic gap in this lexicalization pattern, and has even gone so far as to claim that it is a linguistic universal, i.e. that it can be found in *all* of the world’s languages.⁹ The table above shows that this gappy pattern arises not only in natural languages, but even in the technical jargon of scientific disciplines such as logic.

We now turn to the Aristotelian relations that occur in the square of opposition, i.e. contradiction, contrariety, subcontrariety and subalternation. These relations are classically defined as follows (Parsons 2012): two statements φ and ψ are said to be

| | | |
|----------------------|--|--|
| <i>contradictory</i> | iff φ and ψ cannot be true together | and φ and ψ cannot be false together, |
| <i>contrary</i> | iff φ and ψ cannot be true together | and φ and ψ can be false together, |
| <i>subcontrary</i> | iff φ and ψ can be true together | and φ and ψ cannot be false together, |
| <i>subaltern</i> | iff φ entails ψ | and φ is not entailed by ψ . |

In classical syllogistics, these relations are applied to categorical statements; for example, the statements “all humans are mortal” and “no humans are mortal” are said to be contrary to each other. In contemporary work, the Aristotelian relations are generalized to arbitrary logical systems and lexical fields—e.g. the modal logic S4 and the field of subjective quantification—and their definitions are characterized model-theoretically. For example, φ and ψ being contrary in S4 is defined as $\models_{S4} \neg(\varphi \wedge \psi)$ and $\not\models_{S4} \varphi \vee \psi$; the other relations can be defined entirely analogously (Demey 2012; Demey and Smessaert 2014; Smessaert and Demey 2014a;b; 2015).¹⁰ Both the classic and the model-theoretic approach typically take the Aristotelian relations to hold between between *object language* statements. This is particularly clear in the model-theoretic definition: for example, the condition $\models_{S4} \neg(\varphi \wedge \psi)$ —which occurs in the definition of φ and ψ being S4-contrary—is only meaningful if the formula $\neg(\varphi \wedge \psi)$, and thus also the individual formulas φ and ψ , come from some well-defined object language \mathcal{L}_{S4} .¹¹ The

⁸The asterisk (*) is commonly used in linguistics to express that a given word, phrase or sentence is not correct in a given language (in the current case: English).

⁹The discrepancy can even be found in Latin (A: *omnis*, I: *aliquisnullus*, E: *nullus* versus O: **nomnis*), as was already pointed out by Thomas Aquinas in the 13th century (Horn 1989, 253).

¹⁰As usual, $\models_{S4} \alpha$ means that α is true in all states of all reflexive transitive Kripke models.

¹¹As a consequence of the model-theoretic approach, the Aristotelian relations are also extremely sensitive to the specific details of the underlying logical system (Demey 2015; Demey and Smessaert

same remark also holds for the classical definition, if the word “true”—which occurs in the conditions “can[not] be *true* together”—is interpreted as “true in a state in a reflexive transitive Kripke model” (recall Footnote 10). However, the classical definition can also be interpreted strictly informally (i.e. without making any reference to models of a logical system), and in that case, the Aristotelian relations can also be taken to hold between *metalinguage* statements.

Since the students described in Section 2 are only familiar with the classical definition (and not with the contemporary model-theoretic definition) of the Aristotelian relations, they typically experience very little difficulty in generalizing these relations from object language to metalanguage statements. I will now discuss in more detail which Aristotelian relations hold between the metalanguage statements described earlier in this section.

- A formula φ is either true in at least one model, or in no models at all. In the first case, it is true that φ is satisfiable and false that φ is a contradiction; in the second case, it is false that φ is satisfiable and true that φ is a contradiction. This shows that the statements “ φ is a contradiction” and “ φ is satisfiable” cannot be true together and cannot be false together, i.e. these statements are *contradictory* to each other. In exactly the same way, the statements “ φ is a tautology” and “ φ is a non-tautology” can be shown to be *contradictory* to each other as well.
- If φ is true in all models, then (assuming that the logical system has any models at all) φ is true in at least one model. However, if φ is contingent, then φ is true in at least one model without being true in all models. This shows that there is a *subalternation* from “ φ is a tautology” to “ φ is satisfiable”. In exactly the same way, it can be shown that there is also a *subalternation* from “ φ is a contradiction” to “ φ is a non-tautology”.
- Consider an arbitrary model (assuming that the logical system has any models at all). The formula φ is either true or false in this model. In the first case, it is not true that φ is a contradiction; in the second case, it is not true that φ is a tautology (\dagger). However, if φ is contingent, then φ is true in at least one model (so it is false that φ is a contradiction) and false in at least one model (so it is false that φ is a tautology) (\ddagger). Together, (\dagger) and (\ddagger) show that “ φ is a tautology” and “ φ is a contradiction” cannot be true together, but can be false together, i.e. these statements are *contrary*. In exactly the same way, “ φ is satisfiable” and “ φ is a non-tautology” can be shown to be *subcontrary*.

In summary, the four metalogical notions introduced at the beginning of this section and the Aristotelian relations holding between them constitute a square of

2017). Consider, for example, the normal modal logics D, $D + \{\diamond\varphi \rightarrow \Box\varphi\}$ and K (Hughes and Cresswell 1996). The formulas $\Box p$ and $\Box\neg p$ are contrary in the first system, contradictory in the second, and independent in the third.

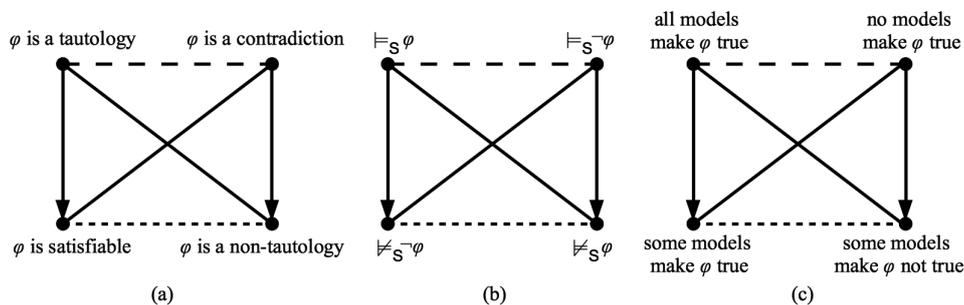


Figure 1: Aristotelian squares for (a) metalogical notions, (b) their symbolic abbreviations, and (c) their categorical definitions.

opposition. Figure 1 shows three different variants of this metalogical square.¹² The entire analogy between metalogic and syllogistics that has been described in this section is epitomized by this metalogical square.

It should be emphasized that the idea of introducing and explaining a new cluster of ideas by showing how it can be used to decorate a square of opposition is certainly not new. For example, Burgess-Jackson (1998) describes how he teaches philosophy of law by showing that four of the most important legal-theoretical theories constitute a square of opposition. The metalogical square presented here is obtained by taking the subject term S = “model of the logical system S ” and the predicate term P = “makes φ true”; completely analogously, Burgess-Jackson’s legal-theoretical square is obtained by taking S = “legal system” and P = “system in which legality is a function of morality”.

4 Pedagogical Uses of the Metalogic/Syllogistics Analogy

In the previous section I have described an analogy between metalogic and syllogistics, which is summarized by the metalogical squares of oppositions shown in Figure 1. I will now discuss a number of ways in which this analogy can be used to teach metalogic more effectively, especially in pedagogical contexts similar to those described in Section 2.

First of all, the analogy provides students with a concrete way to grasp new, metalogical ideas and integrate them with their previously acquired knowledge of syllogistics. For example, (the definitions of) unfamiliar notions such as ‘tautology’ and ‘satisfiability’ turn out to be nothing more than just specific instances of A- and I-type categorical statements from syllogistics. These kinds of integrative insights should help to maximize students’ comprehension and retention of the metalogical material. Furthermore, the analogy expresses a strong continuity

¹²Figure 1 adopts a common convention for visualizing the Aristotelian relations, viz. solid lines (—) for contradiction, long dashed lines (---) for contrariety, short-dashed lines (- - -) for subcontrariety, and arrows (→) for subalternation.

between ‘object-level logic’ and ‘meta-level logic’, and might thus prove helpful in attenuating students’ perception of these two areas as radically different and unrelated parts of logic.

The previous remarks apply not only to the individual notions, but also to the relations holding between them. For example, in order to argue for the subalternation from φ being a tautology to φ being satisfiable, one has to invoke the assumption that the logical system has any models at all, i.e. that the system is (*semantically*) *consistent*. Students who are familiar with syllogistics, however, quickly identify this as the well-known assumption of *existential import*, i.e. the assumption that the subject terms of categorical statements should not be ‘empty’. This assumption is itself not some fifth categorical statement that is explicitly present in the square of opposition; rather, it is an implicit precondition for the Aristotelian relations holding between the statements that *are* present in the square. This helps students to intuitively appreciate that the question whether a given logical system is consistent is more fundamental than the question which of its formulas are tautologies, which are satisfiable, etc.

Another potential application concerns students’ understanding of the properties of \models_S . It is an easy exercise to show that $\neg(\varphi \wedge \neg\varphi)$ and $\varphi \vee \neg\varphi$ are tautologies. The first of these has a straightforward metalogical analogue: it can never simultaneously be the case that $\models_S \varphi$ and $\models_S \neg\varphi$ (again, assuming that S has any models at all). However, students are often misled into thinking that the second tautology also has a metalogical analogue, and erroneously claim that it must always be the case that $\models_S \varphi$ or $\models_S \neg\varphi$. A helpful response to this claim might consist in pointing out that $\models_S \varphi$ and $\models_S \neg\varphi$ are not contradictories, but merely contraries: while they cannot be true together (see the first metalogical analogue), they *can* be false together, i.e. there exist formulas φ such that neither $\models_S \varphi$ nor $\models_S \neg\varphi$. Now, it might be objected that one does not strictly *need* the Aristotelian terminology of contradiction and contrariety to see that it is wrong to claim that always $\models_S \varphi$ or $\models_S \neg\varphi$; after all, this insight can also be gained by reasoning about the definition of \models_S directly. However, this objection fails to take into account that for students who are just getting acquainted with these notions, it might be helpful to be able to rely on their previously acquired knowledge, such as the fact that contrary statements can be false together. Furthermore, pointing out that two notions are not contradictories but merely contraries is a widespread dialectical move in all areas of philosophy,¹³ and even in academic discourse in general,¹⁴ and is thus far more

¹³Here are some random examples from ethics, philosophy of language, phenomenology and metaphysics, respectively: “trust and distrust are not contradictories, but contraries” (Brenkert 1998, 198); “a semifactual and the corresponding counterfactual are not contradictories but contraries, and both may be false” (Goodman 1947, 115); “if we examine these two kinds of connection phenomenologically, as nodi of meaning, we recognize them to be not contradictories but contraries” (Wheelwright 1942, 516); “it might be suggested that rest and motion [...] are not contradictories, but contraries” (Priest 2014, 136).

¹⁴For example, President George W. Bush’s famous remark that “you’re either with us or against us” became an instant classic in critical thinking textbooks, in which it is analyzed as fallacious, since it presents the positions ‘with the USA’ and ‘against the USA’ as contradictories, while they

likely to lead to genuine student understanding than purely formal arguments.

A final pedagogical use is based on the fact that the metalogical square in Figure 1 is a *diagram*, i.e. a visual representation of textual/symbolic information. The usefulness of good visualizations in logic teaching has long been recognized; for example, Jaspars and Velázquez-Quesada (2011, 149) write that “visualization is an important tool for learning formal logic”.¹⁵ Many syllogistic terms are directly related to visual aspects of the square. For example, the relation of *sub*-contrariety is so-called because it occurs in the *lower* part of the square. Furthermore, the laws of subalternation are typically described using visual terms: “truth goes down, falsity goes up” (de Pater and Vergauwen 2005, 101). This diagrammatic nature of the square of opposition can be exploited to further develop the skillful manipulation of the metalogical notions. For example, consider again the statements $\models_S \varphi$ and $\models_S \neg\varphi$ that were discussed in the previous paragraph. There, I argued that students can achieve a deeper understanding of these statements (and their interrelation) by viewing them as *contraries*. Taking into account the diagrammatic nature of the square, however, students can learn to manipulate these notions even more efficiently: a student might be able to conclude that these statements can be false together, simply by recognizing that they occupy the *top two vertices* of the metalogical square (without first consciously drawing the intermediate conclusion that they are contraries). Needless to say, the successfulness of such nearly ‘associative’ reasoning patterns requires a thorough familiarity with the logical laws and visual design of the square of opposition, which students have typically acquired by making large numbers of relevant exercises in their previous logic courses.

5 Conclusion

In this paper, I have described a specific pedagogical context for an advanced logic course, which involves students who have had little training in logic, except for a thorough introduction to syllogistics. Next, I have presented an analogy between metalogic and syllogistics, and argued that this can help to teach metalogic more effectively in the aforementioned pedagogical context.

As was already hinted at in Footnote 5, research on syllogistics has started extending the square of opposition to larger Aristotelian diagrams, such as hexagons and octagons. A natural question, therefore, is whether these larger Aristotelian diagrams might also be useful in teaching metalogic. A particularly promising candidate is the hexagon first constructed by Jacoby (1950), Sesmat (1951) and Blanché (1966), which contains two additional vertices, viz. the Boolean combinations $I \wedge O$ and $A \vee E$. In the metalogical interpretation, the former corresponds to “ φ is satisfiable and a non-tautology”, or, using more widespread terminology,

are actually only contraries, and can thus be false together (one can be neutral: neither with nor against the USA) (Cohen 2009, 69).

¹⁵The visual-diagrammatic properties of Aristotelian diagrams are studied more systematically in Demey and Smessaert (2014; 2016b;c).

“ φ is contingent”; the latter corresponds to “ φ is a tautology or a contradiction”, for which there is no primitive term, but which can be rephrased as “ φ is non-contingent”.¹⁶ The metalogical hexagon thus displays the notions of tautology, contradiction, satisfiability, non-tautology, contingency and non-contingency, and the various Aristotelian relations holding between them.

It would be interesting to investigate whether, and to what extent, this metalogical hexagon is a more powerful pedagogical tool than the metalogical square. For example, the square contains the notion of satisfiability, but not the notion of contingency, and thus has nothing to say about their relation. By contrast, the hexagon does contain both notions, and might thus help to clarify the subtle differences between them, which many students seem to find difficult to grasp.

References

- Béziau, Jean-Yves. 2012. “The Power of the Hexagon.” *Logica Universalis* 6: 1–43.
- Béziau, Jean-Yves. 2013. “The Metalogical Hexagon of Opposition.” *Argumentos* 10:111–122.
- Blanché, Robert. 1966. *Structures Intellectuelles*. Paris: Vrin.
- Brenkert, George G. 1998. “Trust, Business and Business Ethics: An Introduction.” *Business Ethics Quarterly* 8:195–203.
- Burgess-Jackson, Keith. 1998. “Teaching Legal Theory with Venn Diagrams.” *Metaphilosophy* 29:159–177.
- Carrascal, Begoña. 2011. “Teaching Logic in Philosophy.” In Patrick Blackburn, Hans van Ditmarsch, María Manzano, and Fernando Soler-Toscano, editors, *Tools for Teaching Logic 2011*, LNAI 6680, 38–45. Berlin: Springer.
- Cohen, Elliot D. 2009. *Critical Thinking Unleashed*. Lanham, MD: Rowman & Littlefield.
- de Pater, Wim A., and Roger Vergauwen. 2005. *Logica: Formeel en Informeel*. Leuven: Leuven University Press.
- Decker, Jason. 2011. “On Keeping Logic in the Major.” *Teaching Philosophy* 34: 133–142.
- Demey, Lorenz. 2012. “Structures of Oppositions for Public Announcement Logic.” In Jean-Yves Béziau and Dale Jacquette, editors, *Around and Beyond the Square of Opposition*, 313–339. Basel: Springer.

¹⁶Again, this discrepancy in lexicalization ($I \wedge O$ corresponds to a primitive term, but $A \vee E$ does not) is fully in line with empirical work on natural language. Katzir and Singh (2013) and Seuren and Jaspers (2014) have extended Horn’s neo-Gricean theory to incorporate this lexicalization pattern.

- Demey, Lorenz. 2015. “Interactively Illustrating the Context-Sensitivity of Aristotelian Diagrams.” In Henning Christiansen, Isidora Stojanovic, and George Papadopoulos, editors, *Modeling and Using Context*, LNCS 9405, 331–345. Berlin: Springer.
- Demey, Lorenz. 2017a. “Metalogic, Metalanguage and Logical Geometry.” *Submitted*:1–22.
- Demey, Lorenz. 2017b. “Aristotelian Diagrams for Semantic and Syntactic Consequence.” *Manuscript*.
- Demey, Lorenz, and Hans Smessaert. 2014. “The Relationship between Aristotelian and Hasse Diagrams.” In Tim Dwyer, Helen Purchase, and Aidan Delaney, editors, *Diagrammatic Representation and Inference*, LNCS 8578, 213–227. Berlin: Springer.
- Demey, Lorenz, and Hans Smessaert. 2016a. “Metalogical Decorations of Logical Diagrams.” *Logica Universalis* 10:233–292.
- Demey, Lorenz, and Hans Smessaert. 2016b. “The Interaction between Logic and Geometry in Aristotelian Diagrams.” In Mateja Jamnik, Yuri Uesaka, and Stephanie Elzer Schwartz, editors, *Diagrammatic Representation and Inference*, LNCS 9781, 67–82. Berlin: Springer.
- Demey, Lorenz, and Hans Smessaert. 2016c. “Shape Heuristics in Aristotelian Diagrams.” In Oliver Kutz, Stefano Borgo, and Mehul Bhatt, editors, *Shapes 3.0. The Shape of Things*, CEUR-WS 1616, 35–45. Aachen: CEUR.
- Demey, Lorenz, and Hans Smessaert. 2017. “Combinatorial Bitstring Semantics for Arbitrary Logical Fragments.” *Journal of Philosophical Logic*, forthcoming.
- Doyle, John J. 1952. “The Hexagon of Relationships.” *The Modern Schoolman* 29:93–97.
- Englebretsen, George. 1990. “The Myth of Modern Logic.” *Cogito* 4:150–158.
- Faust Jennifer. 2007. “Unreasonable Accommodations? Waiving Formal Logic Requirements for Students with (Relevant) Disabilities.” *Teaching Philosophy* 30:357–381.
- Goodman, Nelson. 1947. “The Problem of Counterfactual Conditionals.” *Journal of Philosophy* 44:113–128.
- Griffith, William B. 1975. “Symbolic Logic and Appraisal of Argument.” *Teaching Philosophy* 1:13–20.
- Hacker, Edward A. 1975. “The Octagon of Opposition.” *Notre Dame Journal of Formal Logic* 16:352–353.

- Hativa, Nira. 2000. *Teaching for Effective Learning in Higher Education*. Dordrecht: Kluwer.
- Horn, Laurence R. 1989. *A Natural History of Negation*. Chicago, IL: University of Chicago Press, 1989.
- Horn, Laurence R. 2012. “Histoire d’*O: Lexical Pragmatics and the Geometry of Opposition.” In Jean-Yves Béziau and Gillman Payette, editors, *The Square of Opposition. A General Framework for Cognition*, 393–426. Bern: Peter Lang.
- Hughes, George E., and Max J. Cresswell. 1996. *A New Introduction to Modal Logic*. London: Routledge.
- Jacoby, Paul. 1950. “A Triangle of Opposites for Types of Propositions in Aristotelian Logic.” *New Scholasticism* 24:32–56.
- Jaspars, Jan, and Fernando R. Velázquez-Quesada. 2011. “Logic in Action.” In Patrick Blackburn, Hans van Ditmarsch, María Manzano, and Fernando Soler-Toscano, editors, *Tools for Teaching Logic 2011*, LNAI 6680, 141–153. Berlin: Springer.
- Jaspers, Dany, and Pieter Seuren. Forthcoming. “The Square of Opposition in Catholic Hands: A Chapter in the History of 20th-Century Logic.” *Logique et Analyse*.
- Katzir, Roni, and Raj Singh. 2013. “Constraints on the Lexicalization of Logical Operators.” *Linguistics and Philosophy* 36:1–29.
- Massey, Gerald J. 1981. “The Pedagogy of Logic: Humanistic Dimensions.” *Teaching Philosophy* 4:303–336.
- Oderberg, David, editor. 2005 *The Old New Logic*. Cambridge, MA: MIT Press.
- Parsons, Terrence. 2012. “The Traditional Square of Opposition.” In Edward N. Zalta, editor, *Stanford Encyclopedia of Philosophy*. Stanford, CA: CSLI Publications.
- Priest, Graham. 2014. *One*. Oxford: Oxford University Press.
- Sesmat, Auguste. 1951. *Logique II. Les Raisonnements. La Syllogistique*. Paris: Hermann.
- Seuren, Pieter, and Dany Jaspers. 2014. “Logico-Cognitive Structure in the Lexicon.” *Language* 90:607–643.
- Smessaert, Hans, and Lorenz Demey. 2014a. “Logical and Geometrical Complementarities between Aristotelian Diagrams.” In Tim Dwyer, Helen Purchase, and Aidan Delaney, editors, *Diagrammatic Representation and Inference*, LNCS 8578, 246–260. Berlin: Springer.

- Smessaert, Hans, and Lorenz Demey. 2014b. "Logical Geometries and Information in the Aristotelian Square of Oppositions." *Journal of Logic, Language and Information* 23:527–565.
- Smessaert, Hans, and Lorenz Demey. 2015. "Béziau's Contributions to the Logical Geometry of Modalities and Quantifiers." In Arnold Koslow and Arthur Buchsbaum, editors, *The Road to Universal Logic*, 475–494. Basel: Springer.
- Struyker Boudier, C.E.M. 1989. *Wijsgerig leven in Nederland en België, 1880-1980, Volume V: De filosofie van Leuven [Philosophical Life in the Netherlands and Belgium, 1880-1980, Volume V: The Philosophy of Leuven]*. Leuven: Leuven University Press.
- Van Evra, James. 1985. "Logic, the Liberal Science." *Teaching Philosophy* 8: 285–294.
- Wheelwright, Philip. 1942. "A Preface to Phenosemantics." *Philosophy and Phenomenological Research* 2:511–519.